Fibonacci Sequences

1, 1, 2, 3, 5, 8, 13, 21, 24

What is the rule for this sequence? Add the two previous numbers to get the next number in the sequence.

Rule Explained:

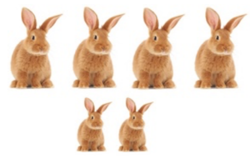
Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits **never die** and that the female **always** produces one new pair (one male, one female) **every month** from the second month on. In other words, rabbits never die, every month each adult pair produces a mixed pair of baby rabbits who mature the next month.

Consider a baby boy rabbit and a baby girl rabbit below

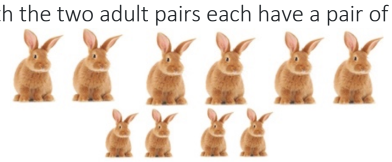
They were fully grown after one month

and did what rabbits do best, so that the next month two more baby rabbits (again a boy and a girl) were born.

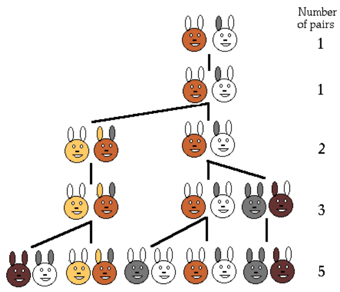
The next month these babies were fully grown and the first pair had two more baby rabbits (again, handily a boy and a girl).



Ignoring problems of in-breeding, the next month the two adult pairs each have a pair of baby rabbits and the babies from last month mature.



Each pair forms the sequence



Therefore, the total number of pairs of rabbits (adult+baby) in a particular month is the sum of the total pairs of rabbits in the previous two months

The number of adult pairs in a given month is the total number of rabbits (both adults and babies)

Real rabbits don't breed as hypothesised, but his sequence still appears frequently in nature, as it seems to capture some aspect of growth. You can find it, for example, in the turns of natural spirals, in plants, and in the family tree of bees. The sequence is also closely related to a famous number called the *golden ratio*.

How many rabbits a single pair can produce after a year with this highly unbelievable breeding process (rabbits never die, every month each adult pair produces a mixed pair of baby rabbits who mature the next month)?

We found that we have a sequence where the next number is found by adding the 2 numbers before it together: 0,1,1,2,3,5,8,13,21,34, …

The rule is $a_n = a_{n-1} + a_{n-2}$

This rule is interesting because it depends on the values of the previous two terms. Rules like this are called “recursive formulae”

The Golden Ratio

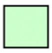
Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 24

Ratios: $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{2}$ $\frac{5}{3}$ $\frac{8}{5}$ $\frac{13}{8}$ $\frac{21}{3}$ $\frac{34}{21}$ $\frac{55}{34}$ $\frac{89}{55}$

What number are the ratios approaching and what is the name for this number?

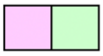
Start with 1 square

1x1



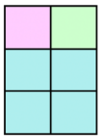
Add a 1x1 square on

1x2



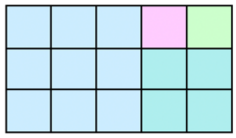
Add a 2x2 square on

3x2



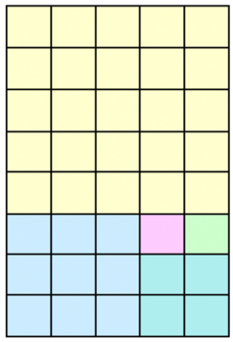
Add a 3x3 square on

3x5



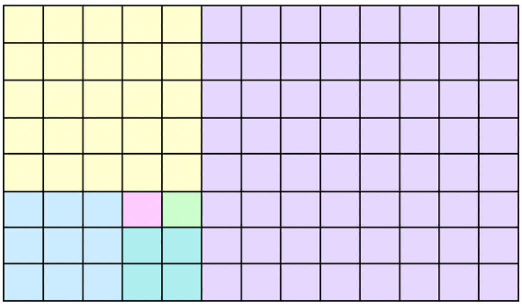
Add a 5x5 square on

8x5



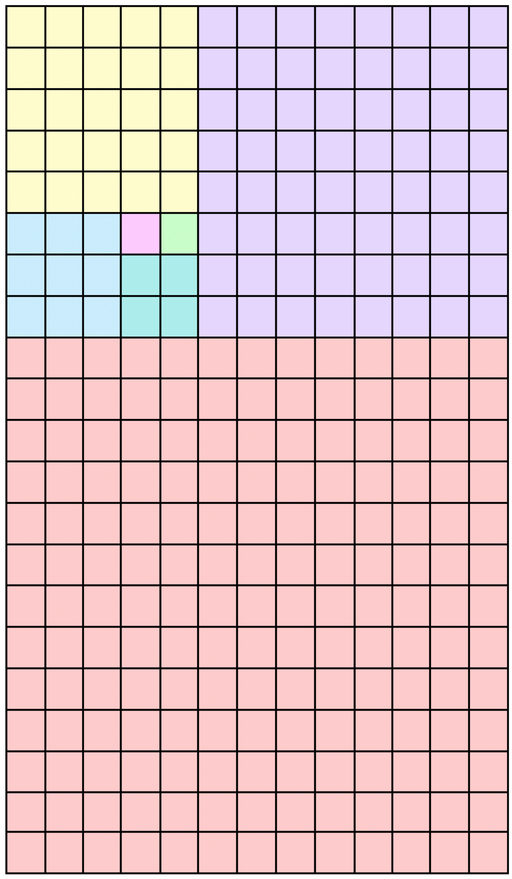
Add an 8x8 square on

8x13

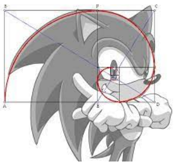
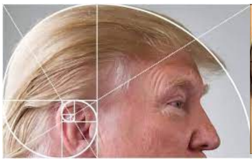
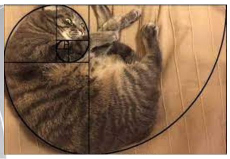


Add a 13x3 square on

21x13



Can you draw the spiral on the image?

Let’s work out some **continued fractions**.

$$1+\frac{1}{1}=\frac{1}{2}$$

$$1+\frac{1}{1+\frac{1}{1}}=\frac{3}{2}$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}=$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}=$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}}}=$$

What number are we approaching?